DESIGN
UNDER UNCERTAINTY

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CHE 4273

Two-Stage Stochastic Optimization Models

Philosophy

- **Maximize the *Expected Value* of the objective over all possible realizations of uncertain parameters.**
- **Typically, the objective is *Profit* or *Net Present Value*.**
- **Sometimes the minimization of *Cost* is considered as objective.**

Uncertainty

- **Typically, the uncertain parameters are: *market demands, availabilities, prices, process yields, rate of interest, inflation, etc.***
- **In Two-Stage Programming, uncertainty is modeled through a finite number of independent *Scenarios*.**
- **Scenarios are typically formed by *random samples* taken from the probability distributions of the uncertain parameters.**

Characteristics of Two-Stage Stochastic Optimization Models

- First-Stage Decisions
  - Taken before the uncertainty is revealed. They usually correspond to structural decisions (not operational).
  - Also called "Here and Now" decisions.
  - Represented by "Design" Variables.
  - Examples:
    - To build a plant or not. How much capacity should be added, etc.
    - To place an order now.
    - To sign contracts or buy options.
    - To pick a reactor volume, to pick a certain number of trays and size the condenser and the reboiler of a column, etc

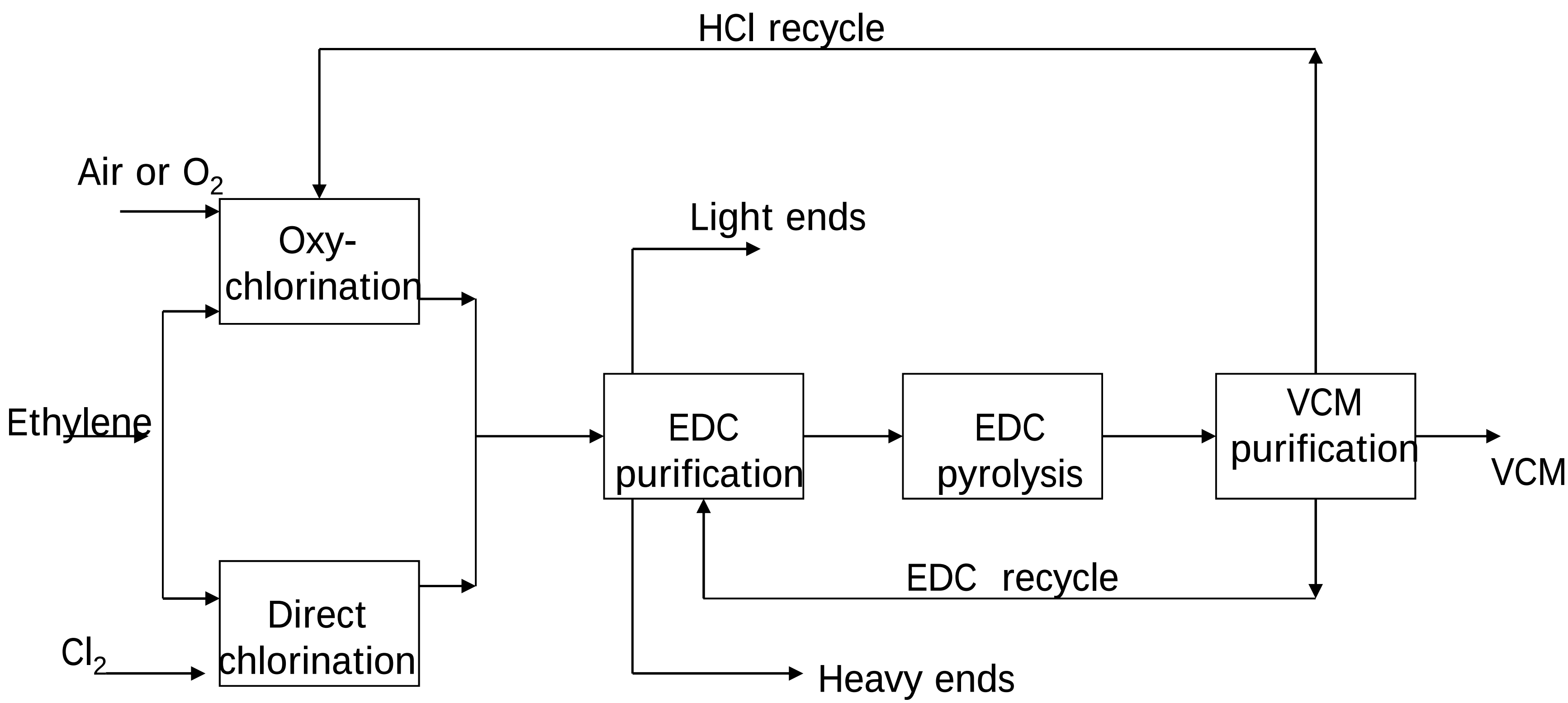
Characteristics of Two-Stage Stochastic Optimization Models

Second-Stage Decisions

- **Taken in order to adapt the plan or design to the uncertain parameters realization.**
- **Also called "Recourse" decisions.**
- **Represented by "Control" Variables.**
- **Example: the operating level; the production slate of a plant.**

- **Sometimes first stage decisions can be treated as second stage decisions. In such case the problem is called a multiple stage problem.**

Example: Vinyl Chloride Plant



Example: Vinyl Chloride Plant

Consider the following forecasts:

Forecasted prices of raw materials product

Year	Ethylene	Chlorine	Oxygen	VCM
	$/ton	$/ton	$/ft3	$/ton
2004	492.55	212.21	0.00144	499.19
2005	499.39	214.14	0.00144	506.19
2006	506.22	216.07	0.00143	513.18
2007	513.06	218.00	0.00142	520.18
2008	519.90	219.93	0.00141	527.18
2009	526.73	221.86	0.00140	529.17
2010	533.57	223.79	0.00139	535.17
2011	540.41	225.72	0.00138	543.17
Std. Dev	24.17	10.56	0.00010	26.15

Forecasted excess demand over current capacity

Year	VCM
	lb-mol/hr
2004	3602
2005	5521
2006	7355
2007	9551
2008	11888
2009	14322
2010	16535
2011	18972

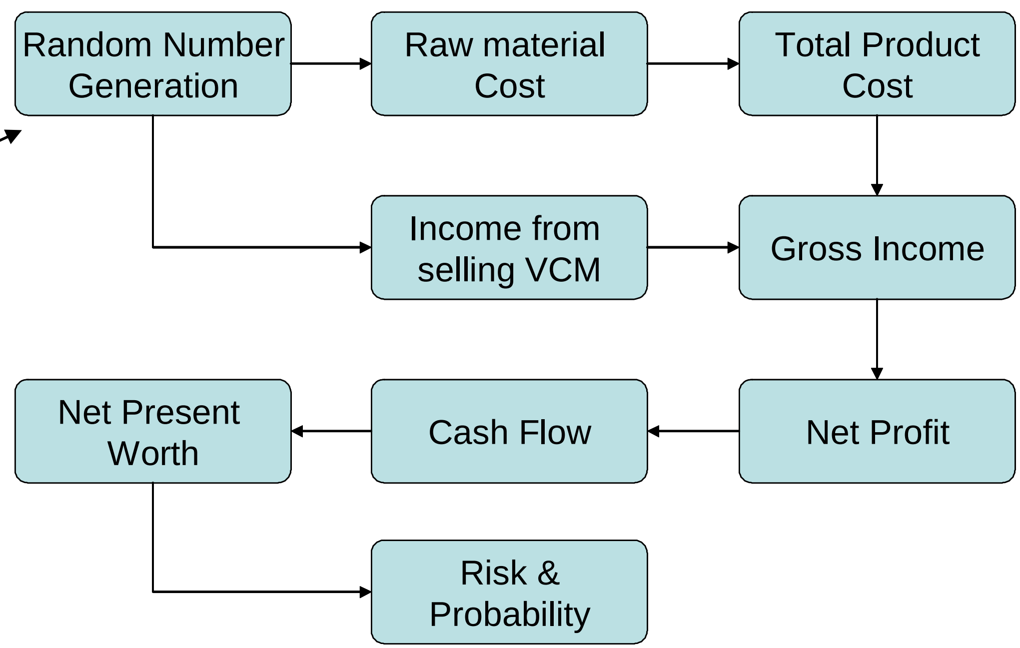
Consider building (in 2004) for three capacities to satisfy excess demand at 2004, 2006 and 2011. Plants will operate under capacity until 2006 or 2011 in the last two cases. These are 3 different first stage decisions.

Example: Vinyl Chloride Plant

The different investment costs are:

Plant Capacity	4090 MMlb/yr	6440 MMlb/yr	10500 MMlb/yr
TCI	$47,110,219	$68,886,317	$77,154,892

Consider the following calculation procedure

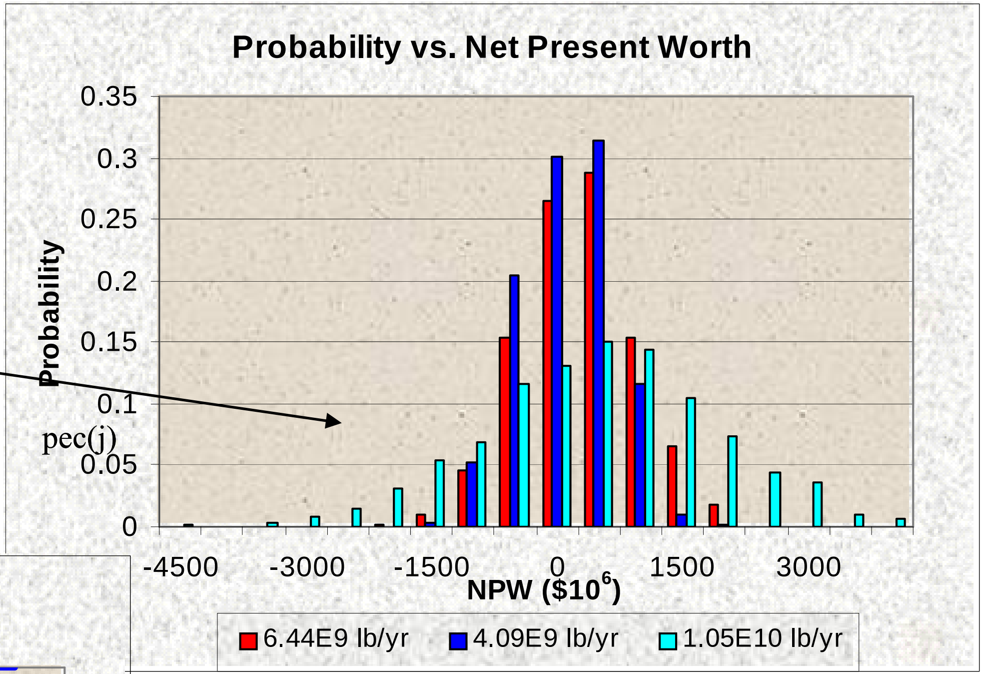


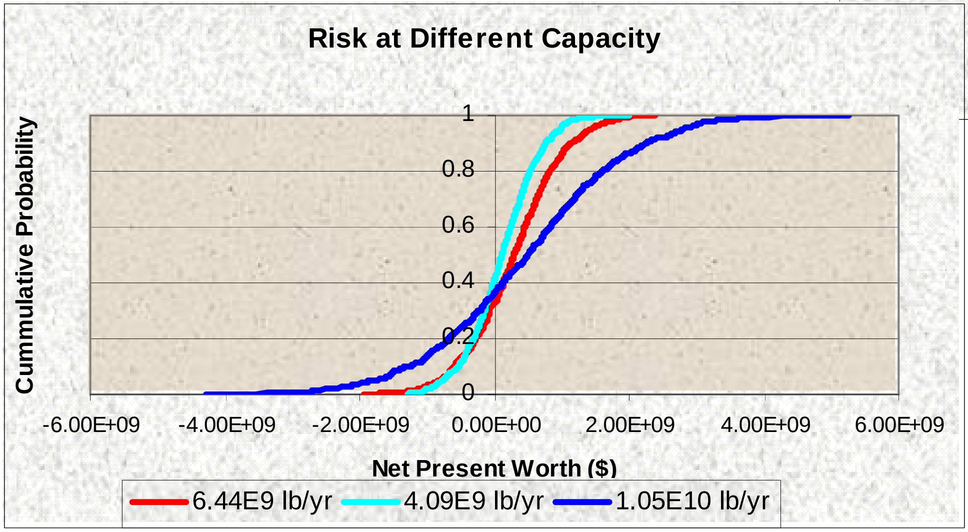
Random numbers are obtained for each year for raw materials and product prices using sampling from a normal distribution. This can be done in Excel

Example: Vinyl Chloride Plant

Histograms and Risk Curves are

Notice the asymmetry in the distributions.





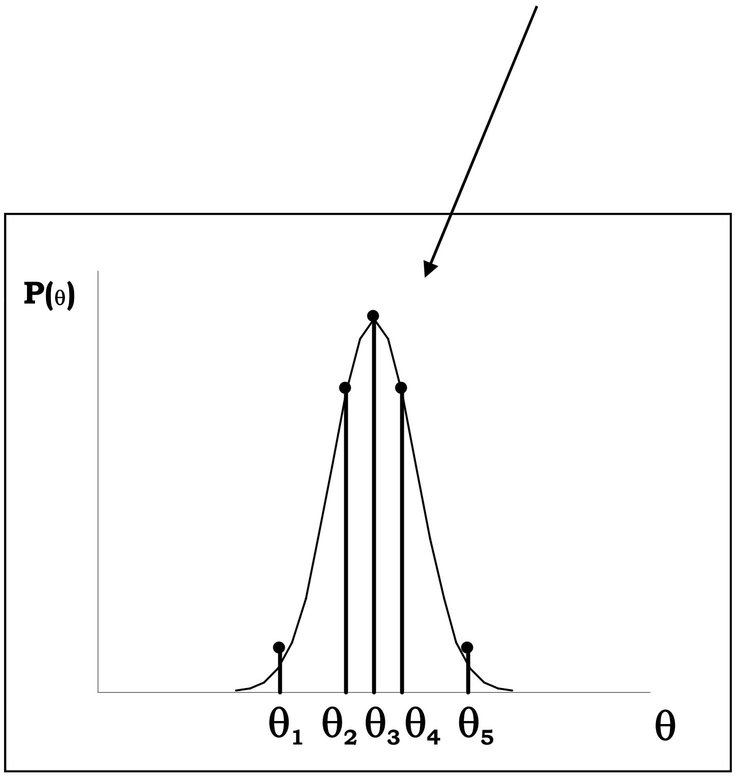
The risk curves show a 36% chance of losing money for the 10.5 billion lbs/year capacity, 31.7% for the 6.44 billion lbs/yr capacity and 41% chance for the 4.09 billion lbs/year capacity.

Expected Profits are: 24%, 25% and 20%.

SCENARIO GENERATION

SCENARIO GENERATION

Consider each parameter's probability distribution.

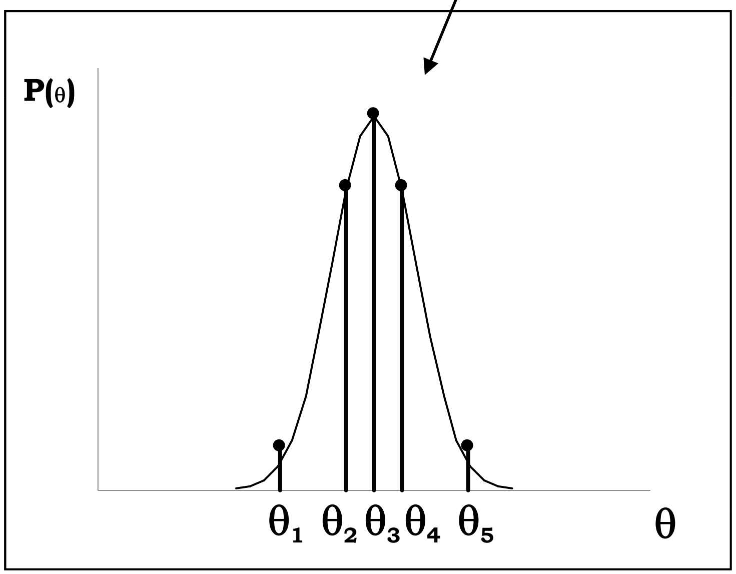


Discretize it.

Option 1: pick values of probabilities. For example, for 3 values, pick 25%, 50% and 25% probability and find the values. Use the cumulative curve to locate the numbers.

SCENARIO GENERATION

Consider each parameter's probability distribution.

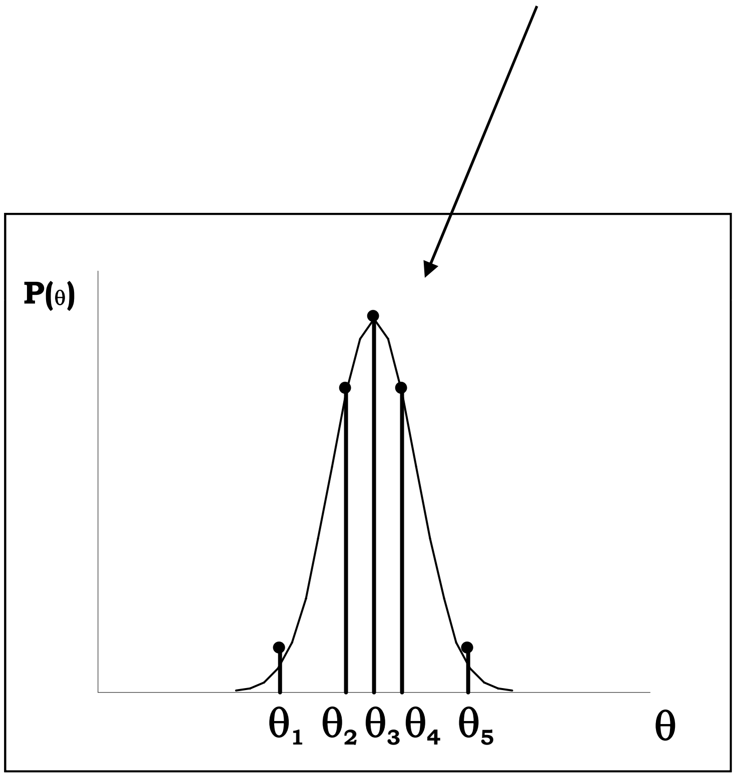


Discretize it.

Option 2: *pick values (equidistantly or randomly) and find the probability that corresponds to them from the area they "span". Use the cumulative curve for this.*

SCENARIO GENERATION

Consider each parameter's probability distribution.



Discretize it.

Option 3: pick equal probability values and find parameter values. For example, for 3 values, pick 33% and locate the points. Use the cumulative curve to do this.

FOR A LARGE NUMBER OF SAMPLES WE USE THIS OPTION

SCENARIO GENERATION

Each scenario is constructed by picking one realization for each parameter.

EXAMPLE:

2 parameters ($\theta_{1,}$ θ_2). If each parameter is discretized in three instances

($\theta_{i,low,}$ 25%, $\theta_{i,avg}$ 50%, $\theta_{i,hig}$ 25%)

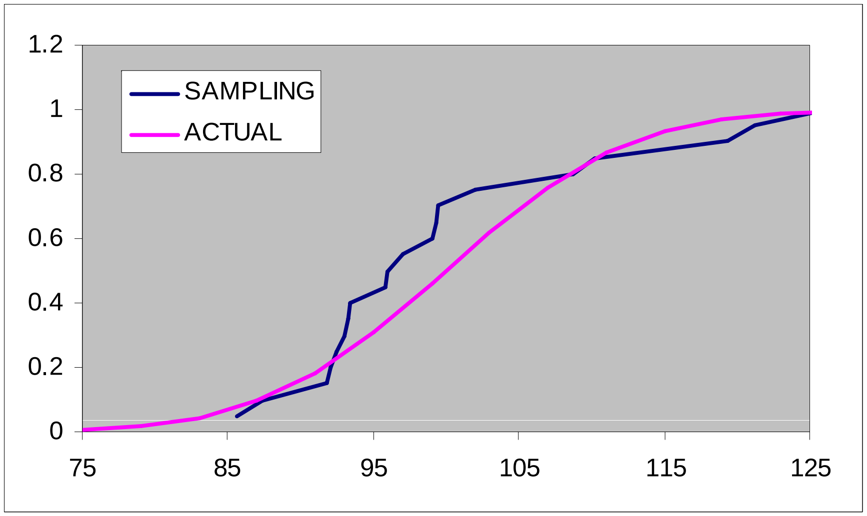
SCENARIO GENERATION

Scenario	Probability	Scenario	Probability
$\theta_{1,low}, \theta_{2,low}$	6.25%	$\theta_{1,hig}, \theta_{2,low}$	6.25%
$\theta_{1,low}, \theta_{2,avg}$	12.5%	$\theta_{1,hig}, \theta_{2,avg}$	12.5%
$\theta_{1,low}, \theta_{2,hig}$	6.25%	$\theta_{1,hig}, \theta_{2,hig}$	6.25%
$\theta_{1,avg}, \theta_{2,low}$	12.5%		
$\theta_{1,avg}, \theta_{2,avg}$	25.0%		
$\theta_{1,avg}, \theta_{2,hig}$	12.5%		

SUM OF ALL PROBABILITIES=1

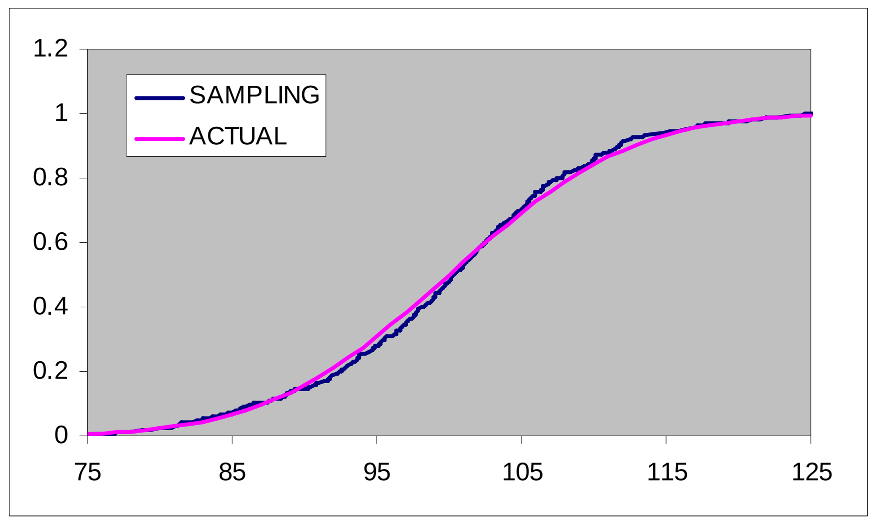
SCENARIO GENERATION

Effect of Small Number of Samples



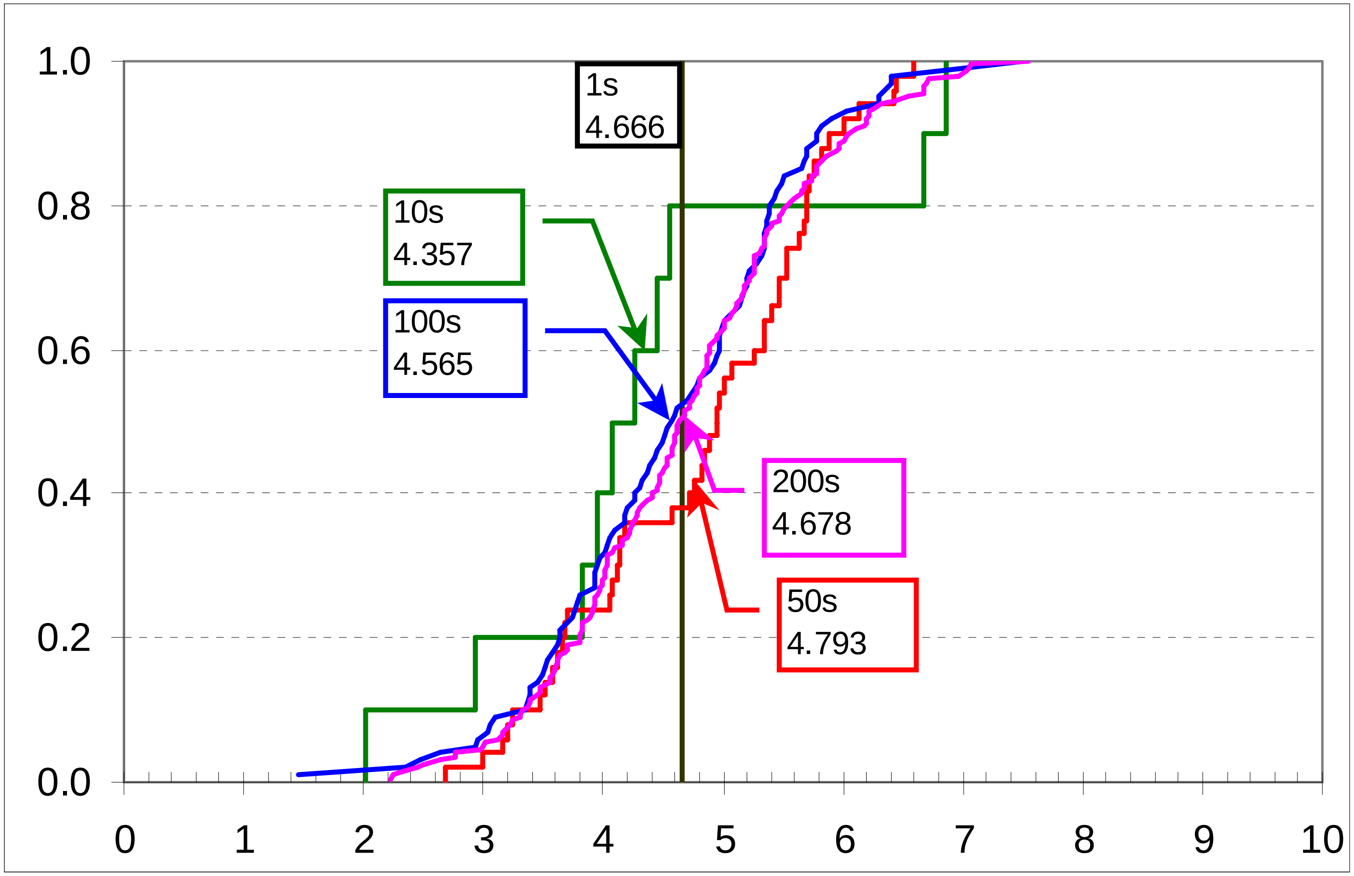
20 Scenarios

500 Scenarios



SCENARIO GENERATION

Effect of the Number of Samples on Results (Gas in Asia)



REGRET ANALYIS

MINIMAX REGRET ANALYSIS

Motivating Example

	s_1 High	s_2 Medium	s_3 Low	Average
A	**19**	**14**	**-3**	**10**
B	**16**	**7**	**4**	**9**
C	**20**	**8**	**-4**	**8**
D	**10**	**6**	**5**	**7**
Max	20(C)	14(A)	5(D)	10(A)

- Traditional way

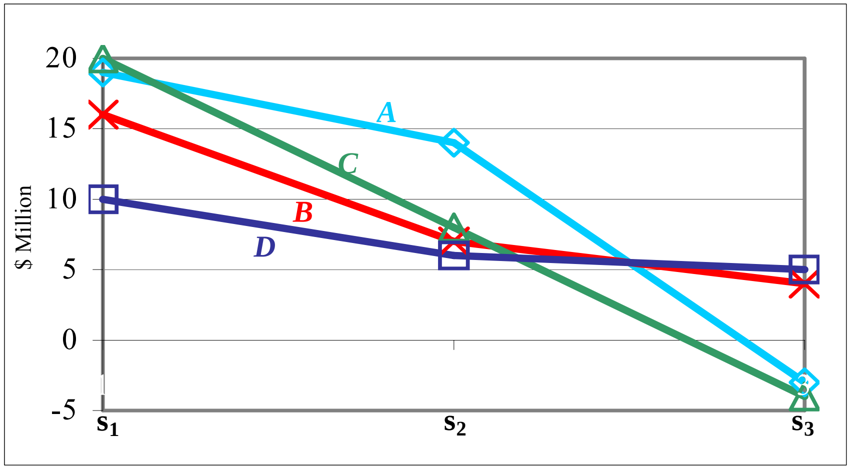
  Maximize Average…select A

- Optimistic decision maker

  MaxiMax … select C

- Pessimistic decision maker

  MaxiMmin … select D



MINIMAX REGRET ANALYSIS

Motivating Example

➢***Calculate regret:***

find maximum regret

➢A … regret = 8 @ low market

➢C … regret = 9 @ low market

➢D … regret = 10 @ high market

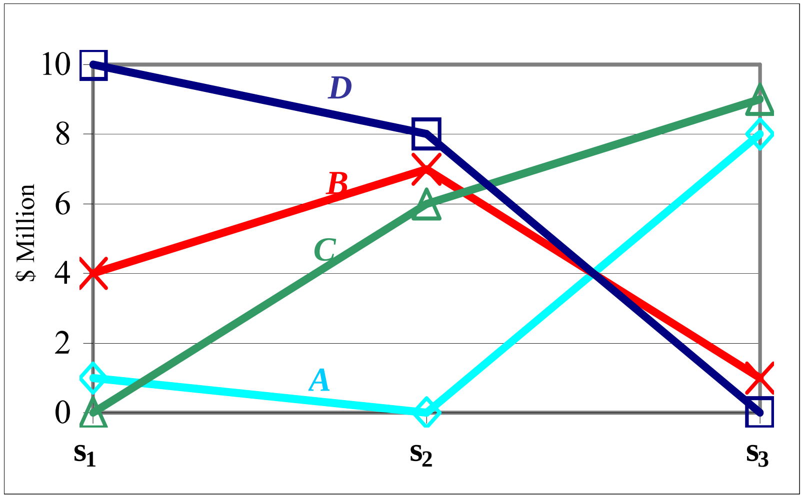
➢B … regret = 7 @ medium market

➢***MINIMAX → B***

➢In general, gives ***conservative*** decision

but not pessimistic.

	s_1 High	s_2 Medium	s_3 Low	Maximum Regret
A	**1**	**0**	**8**	**8**
B	**4**	**7**	**1**	**7**
C	**0**	**6**	**9**	**9**
D	**10**	**8**	**0**	**10**



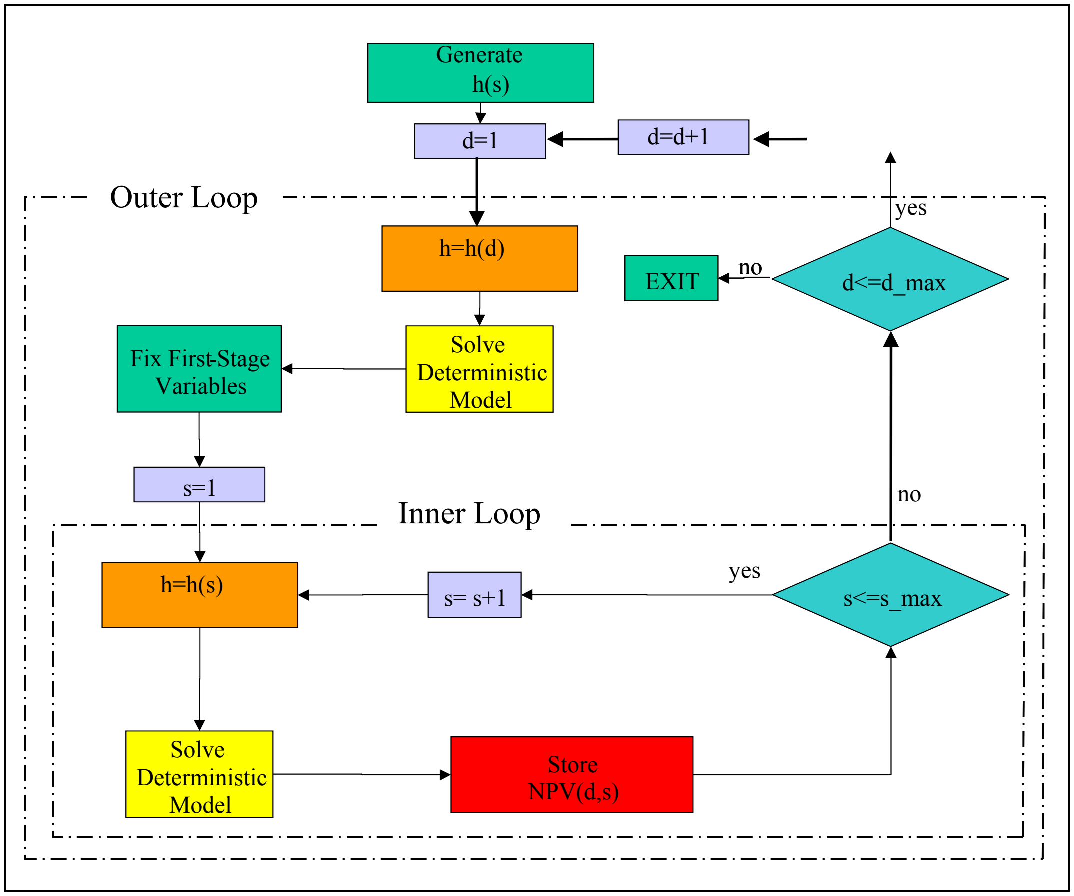
MINIMAX REGRET ANALYSIS

Two-Stage Stochastic Programming Using Regret Theory

NPV								
	s1	s2	s3	s4	s5	ENPV	Max	Min
d1	19.01	10.38	10.57	15.48	10.66	13.22	19.01	10.38
d2	11.15	14.47	8.87	20.54	10.58	13.12	20.54	8.87
d3	12.75	7.81	16.02	22.25	9.16	13.60	22.25	7.81
d4	5.41	9.91	12.63	32.02	8.08	13.61	32.02	5.41
d5	15.09	7.40	8.81	12.48	15.05	11.77	15.09	7.40
Max	19.01	14.47	16.02	32.02	15.05	13.61	32.02	10.38

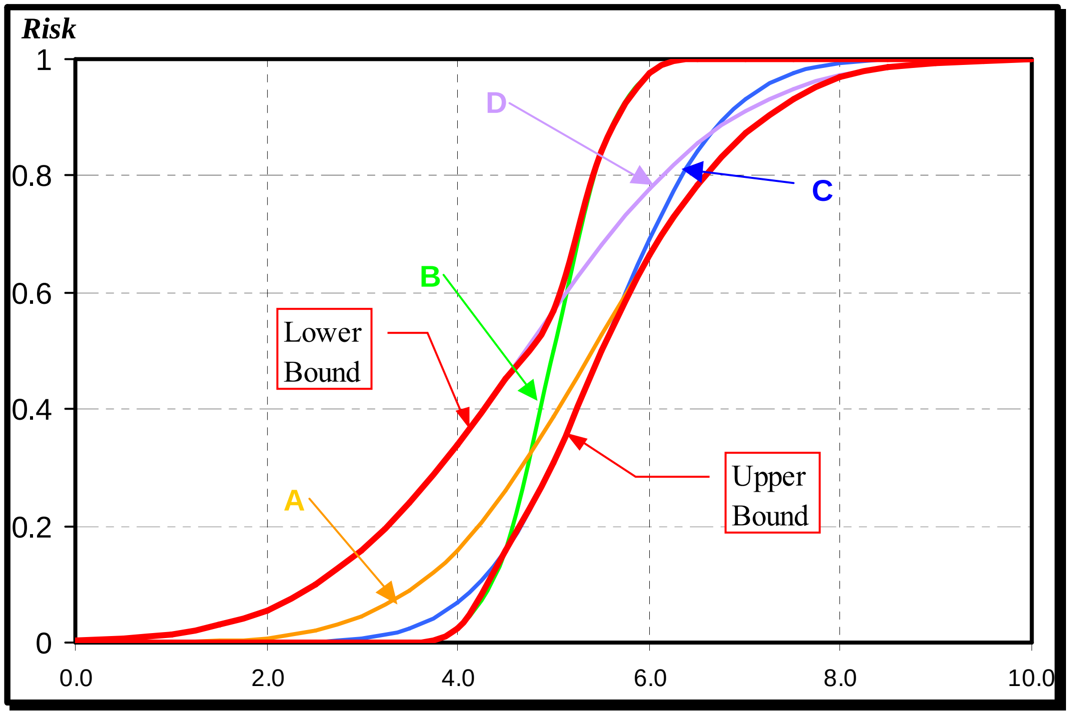
Regret						
	s1	s2	s3	s4	s5	Max
d1	0.00	4.09	5.45	16.54	4.39	16.54
d2	7.86	0.00	7.15	11.48	4.47	11.48
d3	6.26	6.66	0.00	9.77	5.89	9.77
d4	13.60	4.56	3.39	0.00	6.97	13.60
d5	3.92	7.07	7.21	19.54	0.00	19.54
					Min	9.77

SAMPLING ALGORITHM



This generates several solutions

UPPER AND LOWER BOUNDS

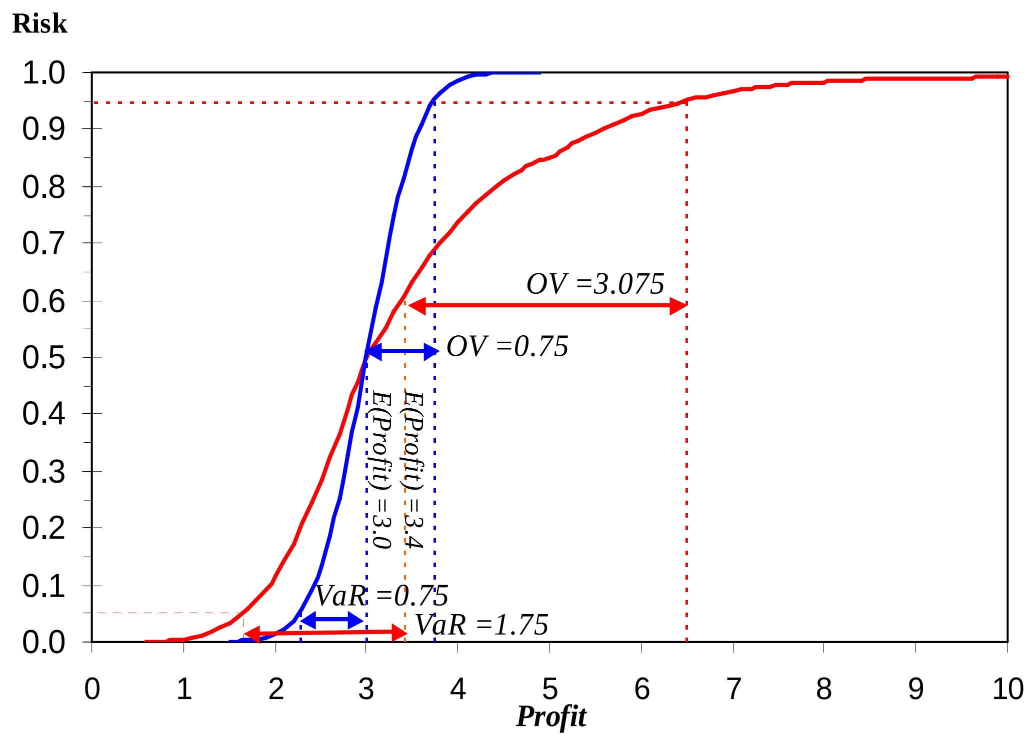


This is very useful because it allows nice decomposition,

That is, there is no need to solve the full stochastic problem

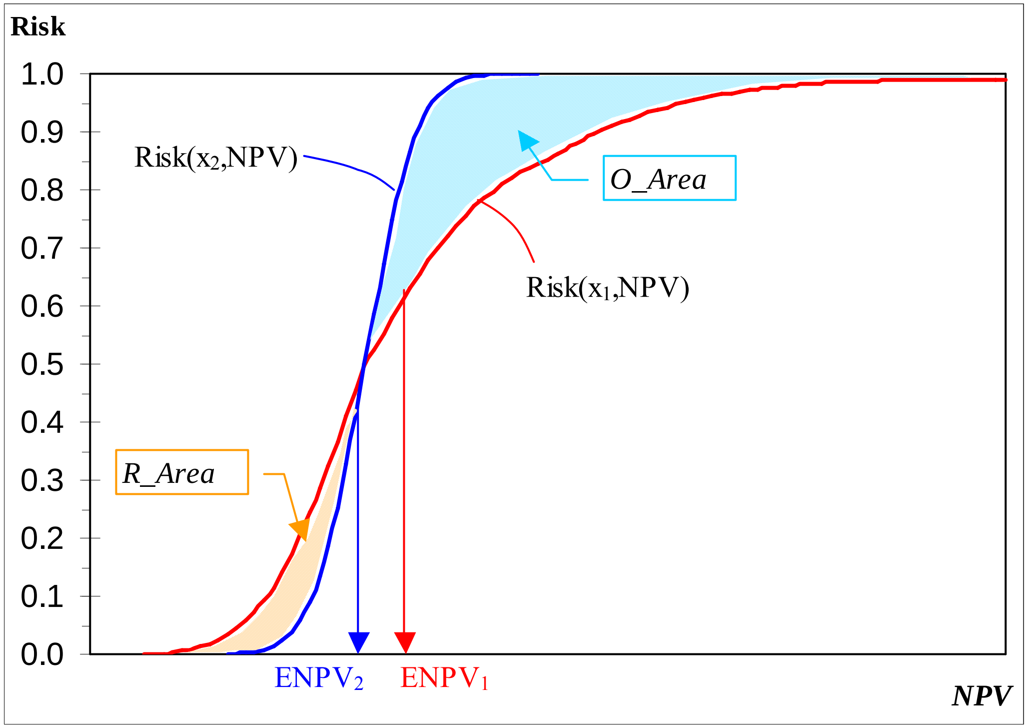
UPSIDE POTENTIAL

Point measure for the upside



AREA RATIO

Comparison measure



CONCLUSIONS

- Regret Analysis can help in identifying good solutions (It can also fail)
- The sampling Algorithm is an important tool to identify upper bounds and good solutions.
- The upper potential is important to be considered.